

Next-to-Leading Order Description of Nucleon Structure Function In Valon Model

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Abstract

We have improved and examined the applicability of the *valon* model where the structure of any hadron is determined by the structure of its constituent quarks. Nucleon structure functions are calculated within this model in the Next-to-Leading order. The results compare well with the experimental data. The model handles the bound state problem and the calculations show a flat or almost flat behavior for F_2 which sets in at some region of $x \leq 10^{-5}$ at fixed Q^2 . The emergence of this behavior is a consequence of the model and was not put in *a priori* as a theoretical guess. It seems that such a flatness can be inferred from HERA data, although, not completely confirmed yet. A set of parton distributions are given and their evolutions are tested. Some qualitative implications of the model for the spin structure of the proton is discussed. **PACS Numbers 13.60 Hb, 12.39.-x, 13.88 +e, 12.20.Fv**

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I. INTRODUCTION

Our understanding of hadrons is based on QCD for the interpretation of Deep Inelastic Scattering (DIS) data together with the spectroscopic description of hadrons in terms of massive constituent quarks. At low energies static properties of the hadrons can be deduced from the latter. The most striking feature of the hadron structure intimately related to their nonvalence quark composition. This substructure can be generated in QCD, however, perturbative approach to QCD does not provide absolute values of the observables and requires the input of non-perturbative matrix elements. Experimental data accumulated during the past ten years has shown that Gottfried Sum Rule is violated, suggesting strong violation of $SU(2)$ symmetry breaking in the nucleon sea [1]. It is possible to resolve the violation of Gottfried some rule by allowing a non-perturbative component to the nucleon sea [2] or in a chiral quark model [3], in the region between the chiral symmetry-breaking scale $\Lambda_\chi \approx 1\text{GeV}$ and the confinement scale $\Lambda_{QCD} \approx 0.1 - 0.3$, hadron can be treated as weakly bound state of effective constituent quarks. The large violation of Ellis-Jaffe sum rule implies that only a small fraction of proton helicity is carried by the quarks, leading to the so-called "SPIN CRISIS". The process of quark evolution produces a large asymmetry for gluon, which at a scale of $10(\frac{\text{GeV}}{c})^2$ can be quite large: $\Delta g \left[Q^2 = 10(\frac{\text{GeV}}{c})^2 \right] \approx 4$, and counter intuitive. In fact, at this scale Δg can be anything; so, something has to compensate for the spin component carried by the radiated gluon in the course of evolution. It turns out that the orbital angular momentum of the produced $q - \bar{q}$ and $q - g$ pairs is the compensating agent which finds a natural place in the constituent picture of the nucleon; provided that a constituent quark is viewed as an extended quasiparticle object composed of a valence quark swirling with a cloud of gluons and sea $q - \bar{q}$ paris. All these fairly successful theoretical attempts suggest the presence of clusters in the nucleon. It seems that there is certain relationships between the constituent quark model of hadron and its partonic structure. Therefore, it is interesting to describe the nucleon structure in terms of these quasiparticle constituent quarks. The picture that emerges is as follows: At high enough Q^2 values it is the structure of the constituent quark that is being probed and at sufficiently low Q^2 , no longer the constituent structure can be resolved. Of course, the above presented picture is not new. In 1974 Altarelli, *et al* pioneered such a model, and more recently, pion structure function was constructed by Altarelli and co-workers in a constituent model [4]. In the early 1980's R.C. Hwa proposed a similar model, the so-called *valon model* [5] which was more elaborate and very successful in analyzing a range of observed phenomena [6] [7]. The rise of F_2 at small- x and larger scale of $Q^2 \geq 5\text{GeV}^2$ posses a Lipatov-type behavior, whereas, at smaller scale of $Q^2 \approx 1 - 3\text{GeV}^2$ it is predicted to develop Regge-type x -dependence; that is, parton distributions and F_2 to be flat or almost flat [8]. This latter prediction although has not been confirmed yet at HERA, but there are some evidences of flattening for F_2 at smallest x -bins in the HERA data [9] as well as EMC and NMC [10] data. In this paper we will utilize the essence of the valon model of Ref.[5] in an attempt to investigate the above mentioned qualitative regularities in the constituent quark model of hadronic structure in the HERA region which is now extended to very low- x values by H1 [9], [11], and ZEUS [12] collaborations. The organization of the paper is as follows; in section II we will give a brief description of the valon model which this work is based on, then we will calculate the nucleon structure in terms of the structure of the valons. In section III, an explicit parameterization

of the parton distributions and the numerical calculations will be outlined; In section IV we discuss some qualitative implications of the model on the spin structure of the nucleon. Finally, in section V, we will discuss the results and elaborate on the concluding results.

II. THE VALON MODEL

The valon model is a phenomenological model which is proven to be very useful in its applied to many areas of the hadron physics. The main features of the model is as follows: (detailed work can be found in Ref. [5] and references therein.) A valon is defined to be a dressed valence quark in QCD with a cloud of gluons and sea quarks and antiquarks. Its structure can be resolved at high enough Q^2 probes. The process of dressing is an interesting subject of its own. In fact, progress is made to derive the dressing process in the context of QCD [13]. A valon is an effective quark behaving as a quasiparticle. In the scattering process the virtual emission and absorption of gluon in a valon becomes bremsstrahlung and pair creation, which can be calculated in QCD. At sufficiently low Q^2 the internal structure of a valon cannot be resolved and hence, it behaves as a structureless valence quark. At such a low value of Q^2 , the nucleon is considered as bound state of three valons, UUD for proton. The binding agent is assumed to be very soft gluons or pions. The constituent picture of hadron also can be based on the Nambu-Jona-Lasinio (NJL) model [14] including the six-fermion $U(1)$ breaking term. In this model, the transition to the partonic picture is described by the introduction of chiral symmetry breaking scale Λ_χ . Nevertheless, for our purpose, combination of all these effects is summarized in finding the parton distributions in the constituent quark and arriving at the nucleon structure functions. One subtle point is that the valons, or constituent quarks, are not free, gluons are also needed. Thus, in addition to valon degree of freedom, gluon degrees are also should be considered. But, it is not known that in an infinite-momentum frame, the valons carry all of the hadron momentum. This concern ultimately ought to be settled by a reliable theory of confinement. As a working hypothesis we shall assume that the valons exhaust the hadron momentum. Let us denote the distribution of a valon in a hadron by $G_{\frac{v}{h}}(y)$ for each valon v . It satisfies the normalization condition

$$\int_0^1 G_{\frac{v}{h}}(y) dy = 1. \quad (1)$$

and the momentum sum rule:

$$\sum_v \int_0^1 G_{\frac{v}{h}}(y) y dy = 1 \quad (2)$$

where the sum runs over all valons in the hadron h . Nucleon structure function $F^N(x, Q^2)$ is related to the valon structure function $f^v(\frac{x}{y}, Q^2)$ by the convolution theorem as follows;

$$F^N(x, Q^2) = \sum_v \int_x^1 dy G_{\frac{v}{N}}(y) f^v(\frac{x}{y}, Q^2) \quad (3)$$

We note that valons are the universal property of the hadron and therefore its distribution is independent of the nature of the probe and Q^2 value. As the Q^2 evolution matrix of F_2

does not depend on the target, the Q^2 evolution of partonic density is the same for partons in a proton or in a valon. It follows then, that if the convolution is valid at one Q^2 , it will remain valid at all Q^2 . Notice that the moments of the parton densities

$$\mathcal{M}(n, Q^2) = \int_0^1 dx x^{n-1} P(x, Q^2) \quad (4)$$

are simply given by the sum of products of the moments

$$M(n, Q^2) = \sum_v \mathcal{M}_{\frac{v}{h}}(n) \mathcal{M}(n, Q^2) \quad (5)$$

At sufficiently high Q^2 , $f^v(\frac{x}{y}, Q^2)$ can be calculated accurately in the leading-order (LO) and Next-to-leading order(NLO) results in QCD and its moments are expressed in terms of the evolution parameter defined by:

$$s = \ln \frac{\ln \frac{Q^2}{\Lambda^2}}{\ln \frac{Q_0^2}{\Lambda^2}} \quad (6)$$

where Λ and Q_0 are the scale parameters to be determined from the experiments. From the theoretical point of view, both Λ and Q_0 should depend on the order of the moments, however, in our approximation we will take them independent of n . Since $f^v(z, Q^2)$ is free of bound state complications which are summarized in $G_{\frac{v}{h}}(y)$; we can describe a U-type valon structure function, say F_2^U as:

$$F_2^U = \frac{4}{9}(G_{\frac{u}{U}} + G_{\frac{\bar{u}}{U}}) + \frac{1}{9}(G_{\frac{d}{U}} + G_{\frac{\bar{d}}{U}} + G_{\frac{s}{U}} + G_{\frac{\bar{s}}{U}}) + \dots \quad (7)$$

where $G_{\frac{q}{U}}$ are the probability functions for quarks and antiquarks to have momentum fraction z in a U -type valon at Q^2 . Similar expressions can be written for the D -type valon. Structure of a valon, then, can be written in terms of flavor singlet(S) and flavor nonsinglet(NS) components as:

$$F_2^U(z, Q^2) = \frac{2}{9}z \left[G^S(z, Q^2) + G^{NS}(z, Q^2) \right] \quad (8)$$

$$F_2^D(z, Q^2) = \frac{1}{9}z \left[2G^S(z, Q^2) - G^{NS}(z, Q^2) \right] \quad (9)$$

For the electron or muon scattering G^S and G^{NS} in eqs.(8 – 9) are defined as:

$$G^S = \sum_{i=1}^f (G_{\frac{q_i}{v}} + G_{\frac{\bar{q}_i}{v}}) \quad G^{NS} = \sum_{i=1}^f (G_{\frac{q_i}{v}} - G_{\frac{\bar{q}_i}{v}}) \quad (10)$$

For the neutrino and anti-neutrino scattering, similar relations can be written, but we will not present them here since we are mainly concerned with HERA data. In the moment representation we will have;

$$\mathcal{M}_2(n, Q^2) = \int_0^1 dx x^{n-2} F_2(x, Q^2) \quad (11)$$

$$\mathcal{M}_\gamma(n, Q^2) = \int_0^1 dx x^{n-1} G_\gamma(x, Q^2) \quad (12)$$

where, $\gamma = \frac{v}{N}, S, NS$. From these equations, for a nucleon eq.(5) follows:

$$M^N(x, Q^2) = \sum_v M_{\frac{v}{N}}(n) M^v(n, Q^2) \quad (13)$$

Solution of the renormalization group equation of QCD provides the moments of singlet and nonsinglet valon structure functions in the LO and NLO and they can be expressed in terms of the evolution parameter of eq.(6). These moments are given in [15], and [16]. To evaluate nucleon structure function we need the distribution of valons in a nucleon. To proceed, we take a simple form for the exclusive valon distribution

$$G_{UUD}(y_1, y_2, y_3) = N y_1^\alpha y_2^\alpha y_3^\beta \delta(y_1 + y_2 + y_3 - 1) \quad (14)$$

The inclusive distribution can be obtained by double integration over unspecified variables. for example:

$$G_{U/p}(y) = \int dy_2 dy_3 G_{UUD}(y_1, y_2, y_3) = B(\alpha + 1, \alpha + \beta + 2)^{-1} y^\alpha (1 - y)^{\alpha + \beta + 2} \quad (15)$$

The normalization factor is fixed by the requirement that

$$\int G_{U/p}(y) dy = \int G_{D/p}(y) dy = 1 \quad (16)$$

The moments of these inclusive valon distributions are calculated according to equation (12) and they are given by

$$U(n) = \frac{B(a + n, a + b + 2)}{B(a + 1, a + b + 2)} \quad D(n) = \frac{B(b + n, 2a + 2)}{B(b + 1, 2a + 2)} \quad (17)$$

where $B(i, j)$ is the Euler beta function with $a = 0.65$ and $b = 0.35$. After performing inverse Mellin transformation we get for the valon distributions:

$$G_{U/p} = 7.98 y^{0.65} (1 - y)^2 \quad G_{D/p} = 6.01 y^{0.35} (1 - y)^{2.3} \quad (18)$$

III. PARTON DISTRIBUTIONS

To determine the parton distributions, we denote their moments by $M_x(n, s)$, where the subscript x stands for valence quarks u_v and d_v , as well as for the sea quarks, antiquarks and the gluon. The moments are functions of the evolution parameter, s . They are given as,

$$M_u(n, s)_v = 2U(n)\mathcal{M}^{NS}(n, s) \quad M_d(n, s)_v = D(n)\mathcal{M}^{NS}(n, s) \quad (19)$$

$$M_{sea}(n, s) = \frac{1}{2f} [2U(n) + D(n)] [\mathcal{M}^S(n, s) - \mathcal{M}^{NS}(n, s)] \quad (20)$$

$$M_g = [2U(n) + D(n)] M_{gQ}(n, s) \quad (21)$$

Where $\mathcal{M}^S(n, s)$ and $\mathcal{M}^{NS}(n, s)$ are the moments of the singlet and nonsinglet valon structure functions and $M_{gQ}(n, s)$ is the quark-to-gluon evolution function. $U(n)$ and $D(n)$ are the U and D type valon moments, respectively, and are given in the previous section. Calculation of M_x is simple; in what follows, instead, the results are presented in parametric form in the NLO.

In determining the patron distributions, we have used a procedure which is consistent with our physical picture. To assure that we are at a high enough Q^2 value, we first used a set of data at $Q^2 = 12\text{GeV}^2$ from 1992 run of H1 collaboration which covers x interval of $x = [0.000383, 0.0133]$. That is, for a single value of s , or Q^2 , we fit the moments by a sum of beta functions that are the moments of the form:

$$xq_v(x) = a(1-x)^b x^c \quad xq_{sea(gluon)}(x) = \sum_i^3 a_i(1-x)^{b_i} \quad (22)$$

The fit is effective as seen in Figure 1. The parameters a, b, c , a_i, b_i , are further considered to be functions of s , the evolution parameter. For the valence sector :

$$a = a_0 + a_1 s + a_2 s^2 \quad (23)$$

similarly for b , and c . For the non-valence sector we have

$$a_i = \alpha_i + \beta_i \exp(s/\gamma_i) \quad (24)$$

with similar form for b_i . The values for these parameters are given in the appendix. Since the formalism has to include both valence and the sea quarks, it is further required that the valence distributions to satisfy the normalization conditions:

$$\int_0^1 q_{u_v}(x, Q^2) dx = 2 \quad \int_0^1 q_{d_v}(x, Q^2) dx = 1 \quad (25)$$

at all Q^2 values, reflecting the number of valence quark of each type. For the sea quarks distributions we will assume $SU(2)$ flavor symmetry breaking inferred from the violation of the Gottfried sum rule [1]. Implementation of the $\bar{u} < \bar{d}$ in the nucleon sea is those obtained in Ref.[7] where we extracted the ratio:

$$\frac{\bar{u}}{\bar{d}} = (1-x)^{3.6} \quad (26)$$

using low p_T physics in the valon-recombination model. We take $xs(x) = \frac{x(u_{sea}(x) + d_{sea}(x))}{4}$ and $xc(x) = \frac{1}{10}x(u_{sea}(x) + d_{sea}(x))$. These choices are made based on the mass ratios of

the involved quarks. All distributions are referring to the proton. The parameterization of patron distributions are those given in equations (22-24). From the fit to the data at $Q^2 = 12\text{GeV}^2$ we determined the scale parameter Q_0^2 and Λ :

$$Q_0^2 = 0.28\text{GeV}^2 \quad \Lambda = 0.22\text{GeV}. \quad (27)$$

Figure 2 shows the shape of the distributions in equations (22) for the valence and the sea quarks at typical values of Q^2 . Our sea quark distribution, although has a sharp rise at small- x , but also damped very fast as x increases. It appears that there are some evidences in the H1 data at low- x bins and low but fixed Q^2 that supports a flat shape for $F_2(x)$. Figure 3 presents this behavior. Such a flattening of F_2 also is elucidated by Gluck, Reya and Vogt in Ref. [8]. Our results favor a somewhat flat or almost flat behavior which sets in at some $x_0 \leq 10^{-5}$ for low Q^2 . This is attractive in the sense that one may argue that the observed flatness corresponds to the small- x Regge behavior. It is possible to generate the rise of F_2 as x decreases either from the DGLAP evolution in $\ln Q^2$ or from BFKL evolution in $\ln(\frac{1}{x})$; although, due to large partonic densities at low x ; they both must be modified to account for the parton recombination effects. Fortunately, at extreme limit of $x \rightarrow 0$, behavior of parton densities can be calculated analytically, though this limit will not be reached within the kinematic range accessible to HERA. It is evident from Figure 3, that our calculation of the proton structure function at low Q^2 ; $Q^2 \geq 0.4$; and small $x \approx 10^{-5} - 10^{-4}$ gives good agreement with the HERA data. we have included GRV(94) results of [17] for comparison. In figure 4, F_2 is plotted as a function of x for various Q^2 values. In Figure 5, F_2 data is plotted as a function of Q^2 for different x bins. The data now extended over four orders of magnitude both in x and Q^2 and our constituent model NLO calculation agrees well with experimental data. In our calculation for $Q^2 < 4\text{GeV}^2$ we have considered only three flavors whereas for higher Q^2 , four active flavors are taken into consideration. Since our main input in determining parton distribution was HERA data at $Q^2 = 12\text{GeV}^2$; and it is limited to low x interval $[0.000383, 0.0133]$; it requires to check that if a satisfactory result for F_2 emerges for the entire range of x . This is presented in Figure 6 for $Q^2 = 20\text{GeV}^2$ and $x = [0.000562, 0.875]$ with the combined data from H1 Ref.[11], BCDMS, SLAC and EMC taken from the compilation in reference [18]. Functional form of gluon distribution is treated similar to the sea quarks distribution as give in equation (22-24). The pertinent parameters are obtained by imposition of the momentum Sum rule. Unfortunately, there are only a few experimental data points for the gluon distribution, to check against. These data points are the result of a direct measurement of gluon density in the proton at low x [19]. Figure 7, shows the accuracy of our gluon density and the data of Ref.[19], also the GRV results are shown. Finally in figure 8 we plot $\frac{d_v}{u_v}$ for our model and compare it with the world data. Our result for the Gottfried sum rule with four flavors gives

$$S_G = \int_0^1 \frac{dx}{x} [F_2^p - F_2^n] = 0.27. \quad (28)$$

Exactly the same result is obtained from the latest MRST parameterization [20] which is slightly larger than the experimental data of $S_G = 0.235 \pm 0.026$. This discrepancy is originated from the form of $\frac{\bar{q}}{d}$ used in eq.(26). The point here is to demonstrate that a constituent quark model is able to describe the DIS data, even without a fine tuning.

IV. IMPLICATION ON THE NUCLEON SPIN

Spin structure of the nucleon merits a separate consideration of its own. Here we will mention a couple points relevant to the present work.

(i) In the naive quark model the polarized structure function g_2 is zero. However, if we allow quarks to have an intrinsic p_\perp inside the nucleon we can achieve a nonzero value for g_2 :

$$g_2(x) = \frac{1}{2} \sum_q e_q^2 \left(\frac{m_q}{xM} - 1 \right) \Delta q(x) \quad (29)$$

with obvious notations. Neglecting the p_\perp component leads to $m_q = xM$ and hence, $g_2 = 0$ is recovered. In parton model, even such an allocation of p_\perp and getting a nonzero value for g_2 is not free of ambiguity. For, the parton model assumes the validity of impulse approximation and neglects the binding effects of the struck parton in large transverse momentum reactions. Measurement of polarization asymmetries may reveal that they depend on the binding energy. In the model described above, the binding effects are summarized in the constituent quark distributions in the nucleon and the structure of a constituent quark is free of binding problem. In this model p_\perp distribution of quarks within the constituent quark can be obtained by realizing that there exists a size hierarchy: hadron size, constituent quark size, and the point-like partons as stated in [21]. The hadronic structure is determined by the constituent quark wave function and its size is related to low- Q^2 form factor of the nucleon. Constituent quarks have a smaller size and described by their own form factors. Partons are the contents of the constituent quarks and are manifested only in high- Q^2 reactions. In high energy collisions one encounters two scales in p_\perp distribution: the average transverse momentum of pions in multiparticle production processes is about 0.35 GeV whereas, in massive lepton pair production one needs to give a primordial $\langle p_\perp \rangle \approx 0.8$ GeV to the partons in order to describe the data. These two scales are related to the hierarchy of sizes. Pion production is a soft process and its scale is related to the average transverse momentum of the constituent quarks, characterizing the hadronic size. Lepton pair production in $q - \bar{q}$ annihilation is a hard process and its scale is due to the transverse momenta of partons in the constituent quark. Using lepton pair production data, the transverse momentum distribution of quarks in a constituent quark can be parameterized in a Gaussian form

$$p_{\perp q}^c(k^2) = \exp(-1.2k^2) \quad (30)$$

which leads to the ratio of the sizes $\frac{\langle r^2 \rangle_c}{\langle r^2 \rangle_{hadron}} \approx \frac{1}{5}$. Now we can calculate $\langle L_{zq\bar{q}} \rangle$. For a proton of radius $0.85 fm$

$$\langle L_{zq\bar{q}} \rangle = r_c \langle k_\perp \rangle = 0.321$$

(ii) Another point related to the spin content of proton is also relevant here. H. Kleinert [22] was first who suggested to consider the vacuum of massive constituent quark as a coherent superposition of the *Cooper pairs* of massless quarks in analogous to the theory of superconductivity. In the BCS theory, gauge symmetry associated with the particle number conservation is spontaneously broken and the Noether current j_5^μ is not conserved. To restore it, it is realized that there are both collective and single particle excitations. Recently Gaitan [23] has shown that the bare current j_5^μ becomes dressed by a virtual cloud of Goldstone excitations ($q - \bar{q}$ in our case) and the conserved dressed current j^μ is the sum of two parts

$$j^\mu = j_s^\mu + j_{back}^\mu \quad (31)$$

where j_{back}^μ describes the backflow current. This is very similar in our model, where for the dressed constituent quark, the generator of the gauge transformation induces a rotation of the $q\bar{q}$ pair correlations which can be identified as the orbital angular momentum. To this end, the pairing correlation will have the axial symmetry around an anisotropic direction, acting as the local z -axis and the particles forming the cloud of the constituent quark would rotate about this anisotropic direction. what have been said was only a qualitative description; to make it more quantitative, let us consider the spin of a constituent quark, say U . It can be written as

$$J_z^U = \frac{1}{2} = \frac{1}{2}\Delta\Sigma^U + L_{zq\bar{q}} \quad (32)$$

The DIS polarization data suggest that $\Delta\Sigma^p \approx \frac{1}{3}$. Within the $SU(6)$ model [24]

$$\Delta\Sigma^p = (\Delta U + \Delta D)\Delta\Sigma^U = \Delta\Sigma^U \quad (33)$$

comparing with equation (32) we see that $L_{zq\bar{q}} \approx \frac{1}{3}$ i.e. about 70 percent of the spin of a constituent quark is due to the orbital angular momentum of quark pairs in its surrounding cloud, screening the spin of the valence quark:

$$\frac{1}{2}\Delta\Sigma^U = S_{uval.} + S_{sea} = \frac{1}{2} + S_{sea} = \frac{1}{6} \quad (34)$$

resulting in $S_{sea} = -L_{zq\bar{q}} = \frac{1}{3}$. We can take the transverse momentum distribution from eq.(30) and calculate $\langle L_{zq\bar{q}} \rangle$ directly. For the proton of mean radius of $0.86fm$ we will have the mean radius of the constituent quark equal to $0.385fm$ and hence,

$$\langle L_{zq\bar{q}} \rangle = r_c \langle k_\perp \rangle = 0.321$$

which agrees well with the results stated above. Notice that our calculation of J_U and the conclusion reached did not include gluonic effects. In reality we should have included the gluon degree of freedom, however, an inclusion of those effects at the constituent quark level would reduce $(\Delta U + \Delta D)$ by some 30 percent from unity [24], nevertheless the point is clear.

V. CONCLUSION

We have used the valon model to describe the deep inelastic scattering. The model handles the bound state problem and sets the scale parameters. In determining the parton distributions no arbitrary theoretical assumptions are made for low Q^2 . Our parton distributions nicely accommodate the data in a wide range of $x = [10^{-6}, 1]$ and in a broad range of Q^2 spanning from a few GeV^2 up to $5000GeV^2$. Our findings indicate that at $x \leq 10^{-5}$ at fixed Q^2 the structure function F_2 flattens, which may be interpreted as the manifestation of Regge dynamics. As Q^2 , increases, however, the almost flat shape of F_2 gets washed out or pushed towards yet smaller region of x . Thus, we conclude that in a region of $x - Q^2$ plane

the Regge dynamics is in play. This region sets in at some x_0 and not too high Q^2 . Scale violation is obvious from the calculations and the data. The rise of F_2 at $Q^2 < 1 \text{ GeV}^2$ in our model indicates that even at Q^2 as low as a few GeV the evolution has run the course. We further find that certain issues related to the spin structure of the proton can find an interesting place in the framework of the model described.

VI. APPENDIX

In this appendix we give the numerical values for our parton distributions in the NLO both for three and four flavors. These relations are functions of evolution parameter s defined in equation (6). For details see the text.

$f = 4$:

i: For valence sector using Eqs.(22-23) we have:

$$q_v = u_v$$

$$a = 14.132 - 15.759s + 6.795s^2 - 1.082s^3,$$

$$b = 1.608 - 1.206s + 0.37s^2 - 0.0523s^3,$$

$$c = 2.083 + 0.753s + 0.214s^2 - 0.0106s^3,$$

$$q_v = d_v$$

$$a = 14.509 - 5.309s + 2.795s^2 - 0.562s^3,$$

$$b = 1.235 - 1.063s + 0.467s^2 - 0.086s^3,$$

$$c = 2.215 + 0.639s + 0.439s^2 + 0.02s^3,$$

ii: The sea distribution is parametrized as in Eqs. (22-24) with the following values. Anti-quark distributions are followed from Eq.(26) and note thereafter.

$$a_1 = 0.202 + 0.045 \exp(s/0.791),$$

$$b_1 = 3.09 + 4.187 \exp(s/0.538),$$

$$a_2 = -0.064 + 0.152 \exp(s/0.89) ,$$

$$b_2 = 2.574 + 11.156 \exp(s/0.303),$$

$$a_3 = -0.196 + 0.014 \exp(s/0.404),$$

$$b_3 = 3.809 + 1.856 \exp(s/0.669),$$

iii: The gluon distribution is given in Eqs.(22 – 24) with the following numerical values.

$$a_1 = -2.81 + 0.46 \exp(s/0.349),$$

$$b_1 = -4.565 + 8.319 \exp(s/0.498),$$

$$a_2 = -0.948 + 0.297 \exp(s/1.099),$$

$$b_2 = 1.493 + 1.291 \exp(s/0.809),$$

$$a_3 = -0.481 + 0.721 \exp(s/0.948) ,$$

$$b_3 = 0.22 + 1.526 \exp(s/1.306),$$

$f = 3$:

i: For valence sector using Eqs.(22-23) we have:

$$q_v = u_v$$

$$a = 10.891 - 12.525s + 4.87s^2,$$

$$b = 1.457 - 1.18s + 0.384s^2,$$

$$c = 1.979 + 0.254s + 0.537s^2,$$

$$q_v = d_v$$

$$\begin{aligned}
a &= 5.977 - 7.55s + 3.025s^2, \\
b &= 1.095 - 0.44s - 0.067s^2, \\
c &= 2.532 + 0.282s + 0.473s^2,
\end{aligned}$$

ii The sea distribution is parametrized as in Eqs. (22-24) with the following values. Anti-quark distributions are followed from Eq.(26) and note thereafter.

$$\begin{aligned}
a_1 &= 0.31 + 0.064 \exp(s/0.807), \\
b_1 &= 3.905 + 26.187 \exp(s/0.658), \\
a_2 &= -0.064 + 0.023 \exp(s/0.305), \\
b_2 &= 2.644 + 14.156 \exp(s/0.152), \\
a_3 &= -0.063 + 0.023 \exp(s/0.509), \\
b_3 &= 3.508 + 12.856 \exp(s/0.452),
\end{aligned}$$

FIGURES

FIG. 1. F_2^p as a function of x at $Q^2 = 12GeV^2$. Solid line is our results in NLO, dashed line is that of GRV(NLO). Data are from Ref.[11,12]

FIG. 2. Model calculation of parton distributions at various Q^2 .

FIG. 3. Variation of F_2^p as a function of x for $x < 10^{-4}$.

FIG. 4. F_2^p as a function of x for different Q^2 values. Solid line is our results in NLO, dashed line is that of GRV(NLO). Data are from HERA.

FIG. 5. F_2^p as a function of Q^2 for different x -bins. Solid line is our results , dashed line is that of GRV(NLO). Data are from HERA

FIG. 6. F_2^p at $Q^2 = 20GeV^2$ for the entire range of x . Data are from Refs.[11,12,18]

FIG. 7. $xg(x, Q^2)$ at $< Q^2 > = 20GeV^2$ and $f = 4$. Solid line is our results in NLO, dashed line is that of GRV(NLO). Data are from Ref.[19]

FIG. 8. The ratio $\frac{dv}{u_v}$ at $Q^2 = 5GeV^2$ evaluated for four flavors. Data are from CDHS and WA21/25 and Hermes.

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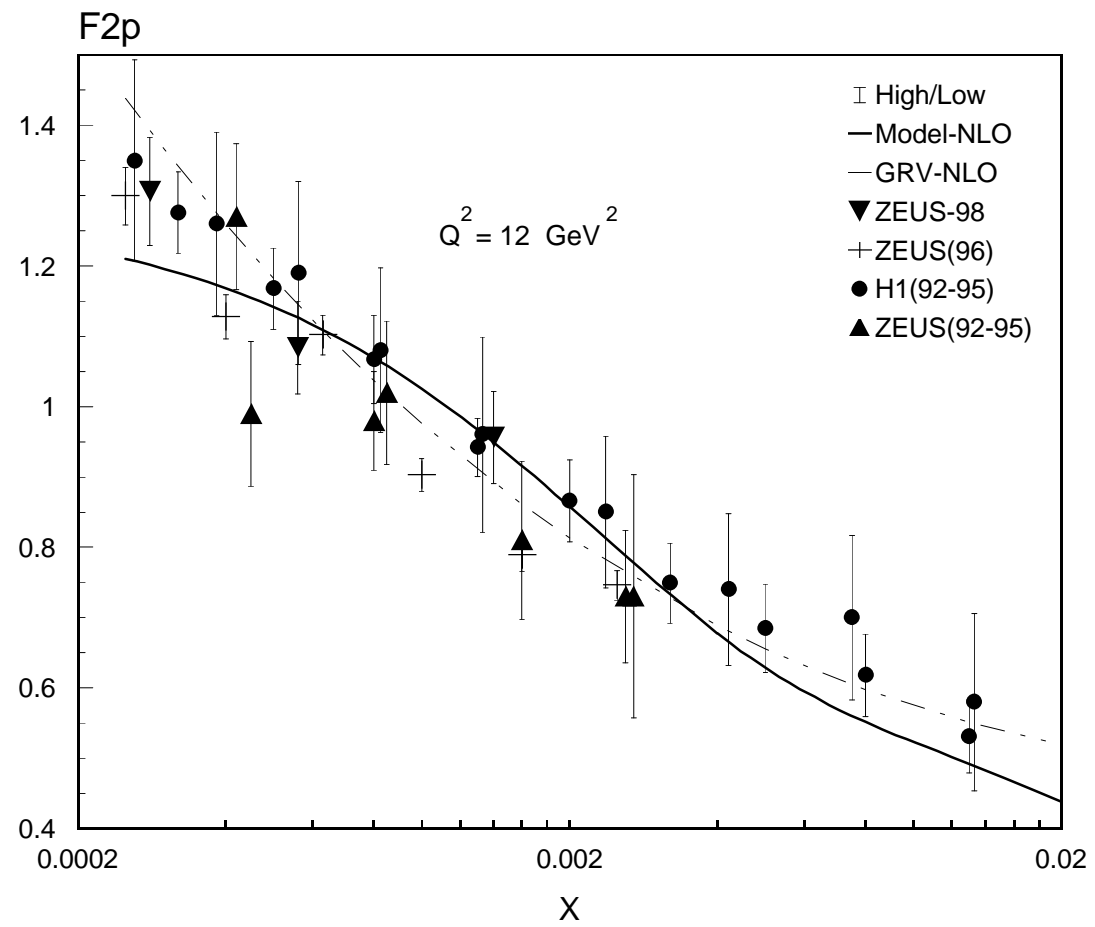


Fig .1

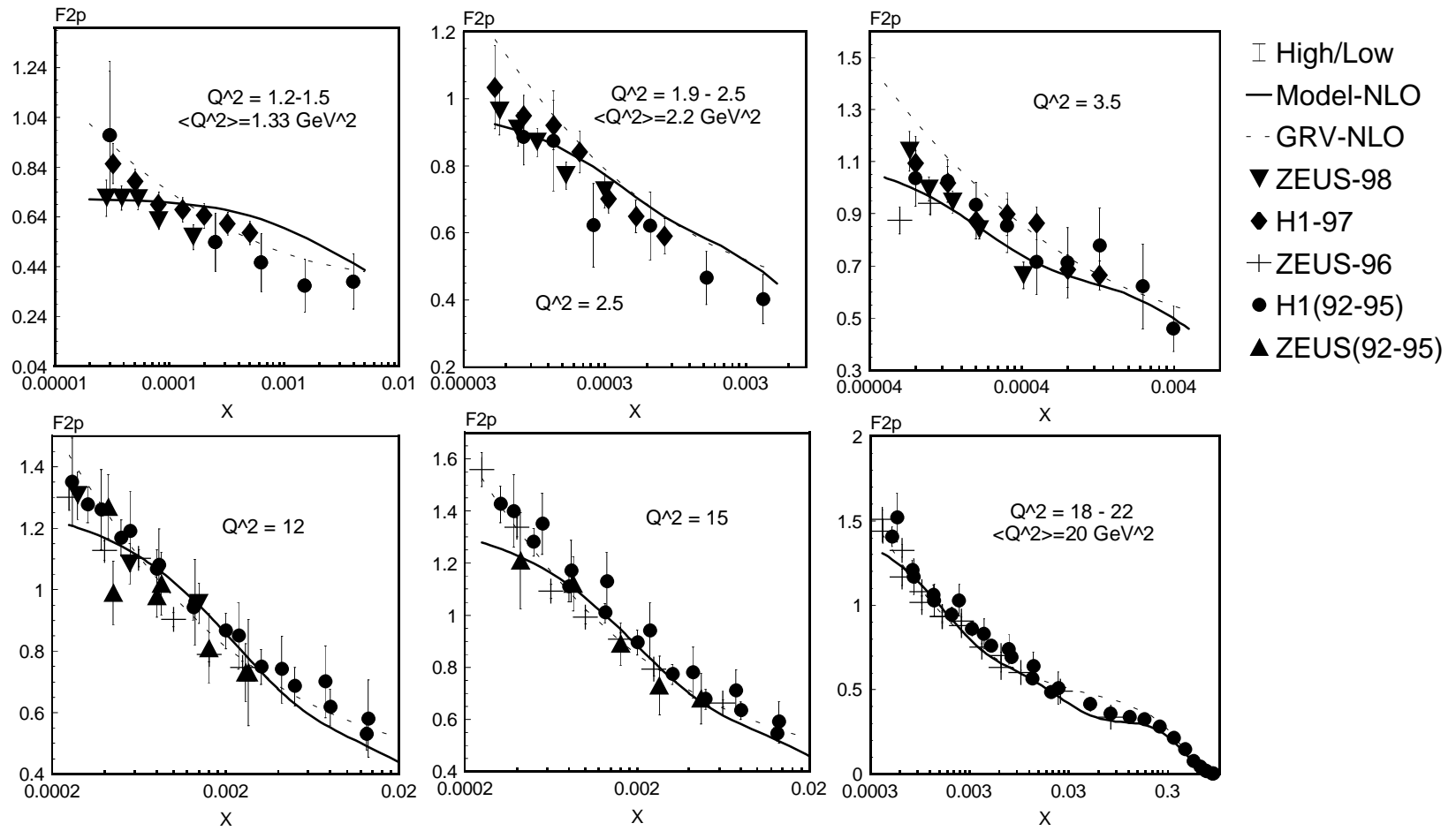


Fig . 4

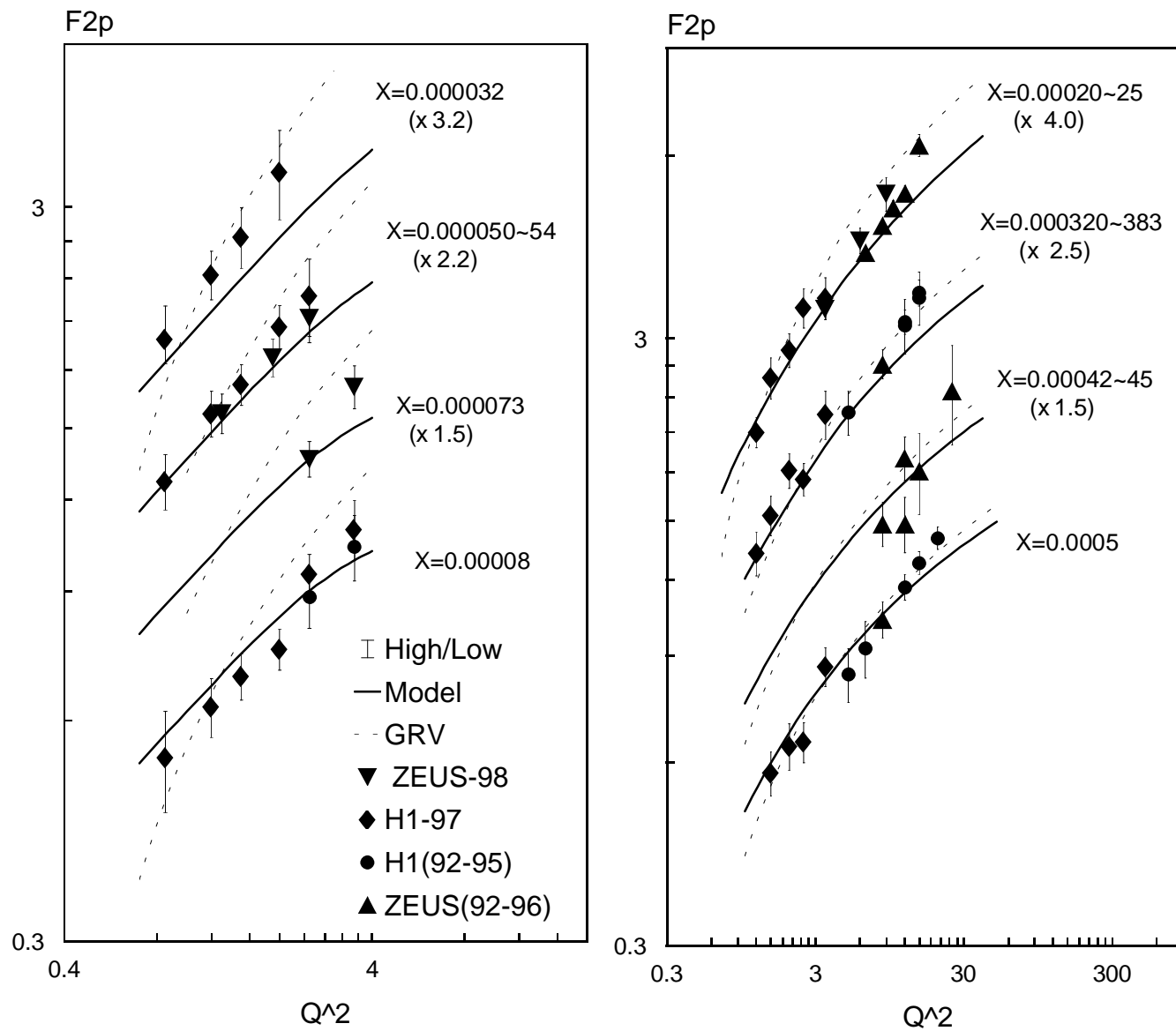


Fig . 5

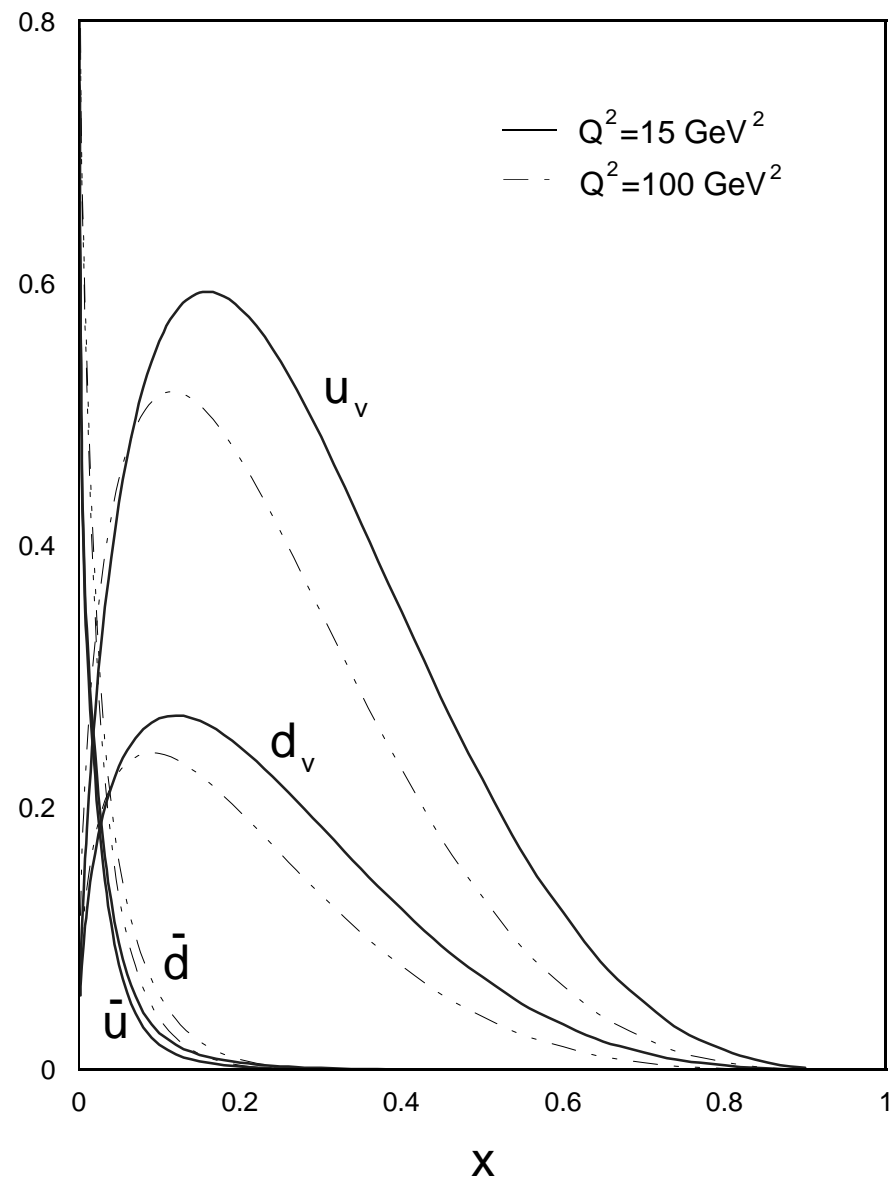


Fig-2

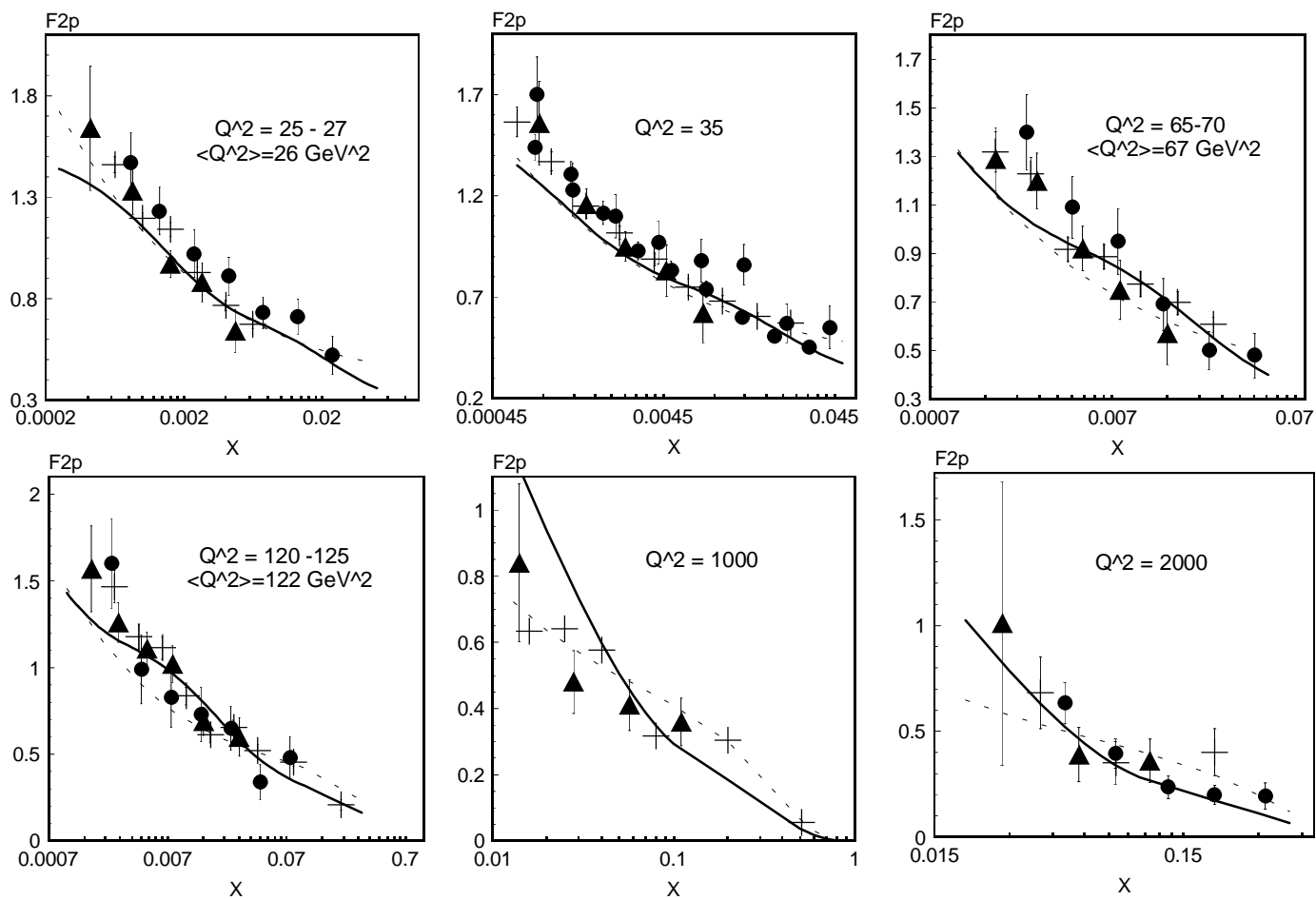


Fig . 4 (continued)

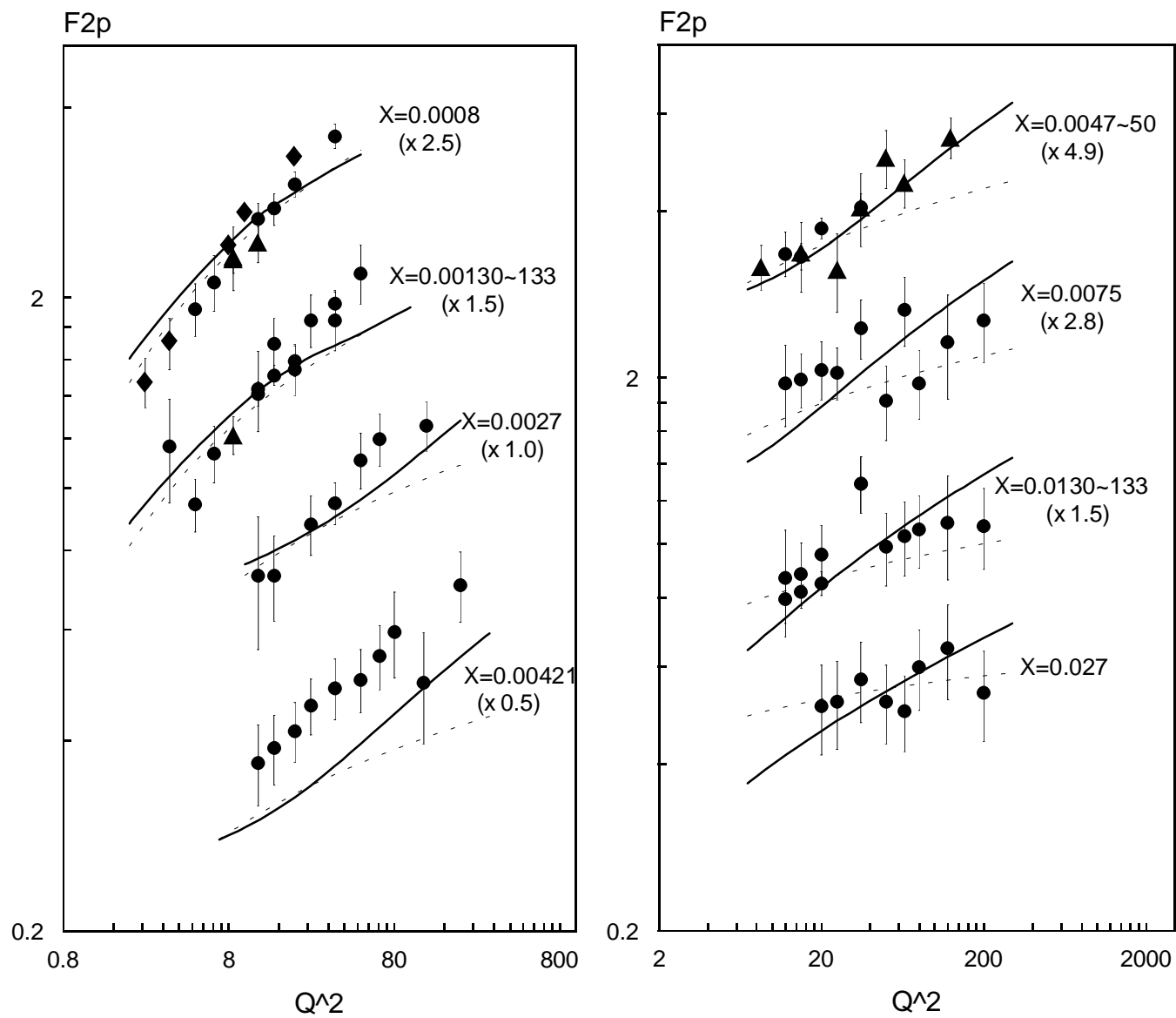


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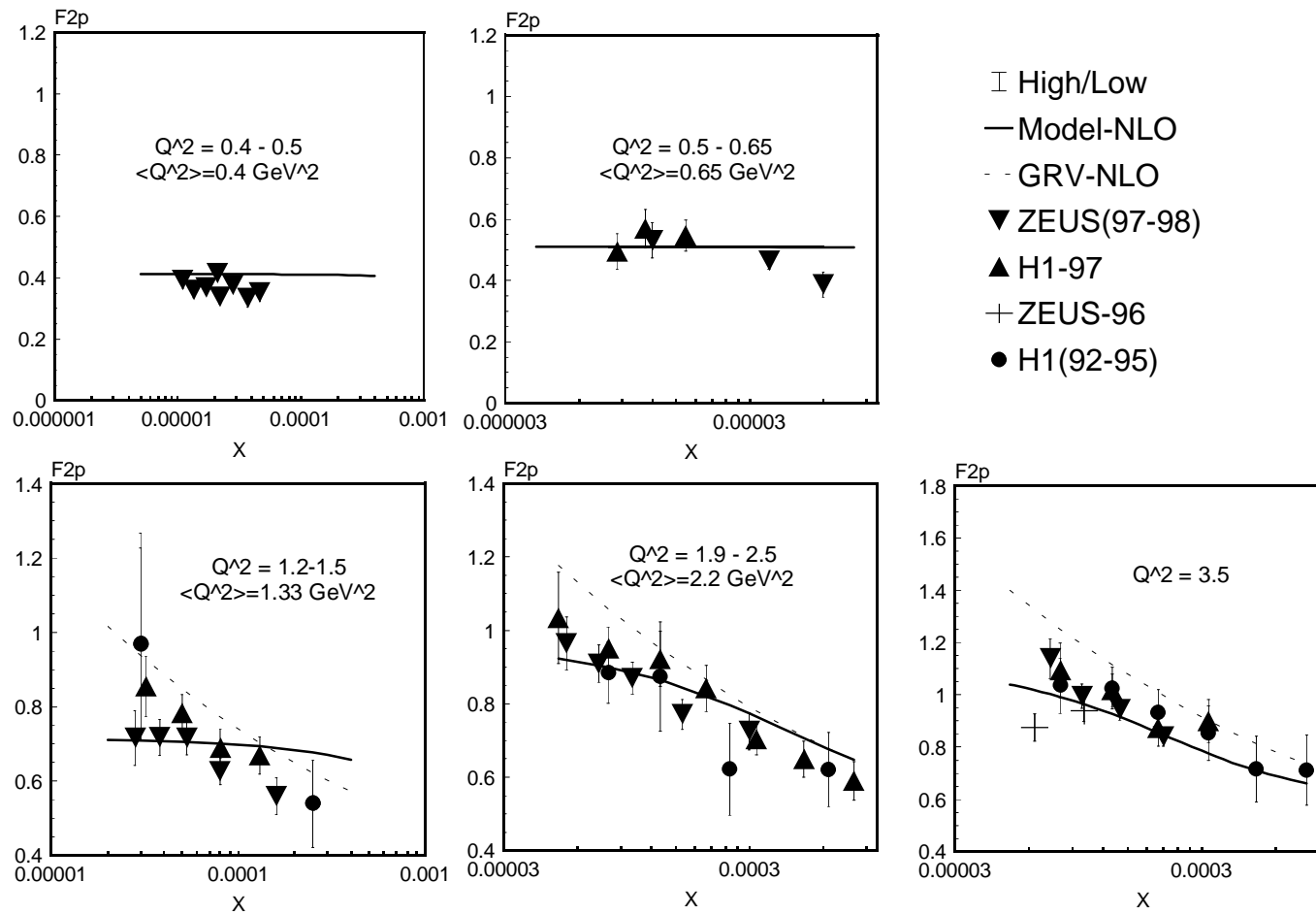


Fig . 3

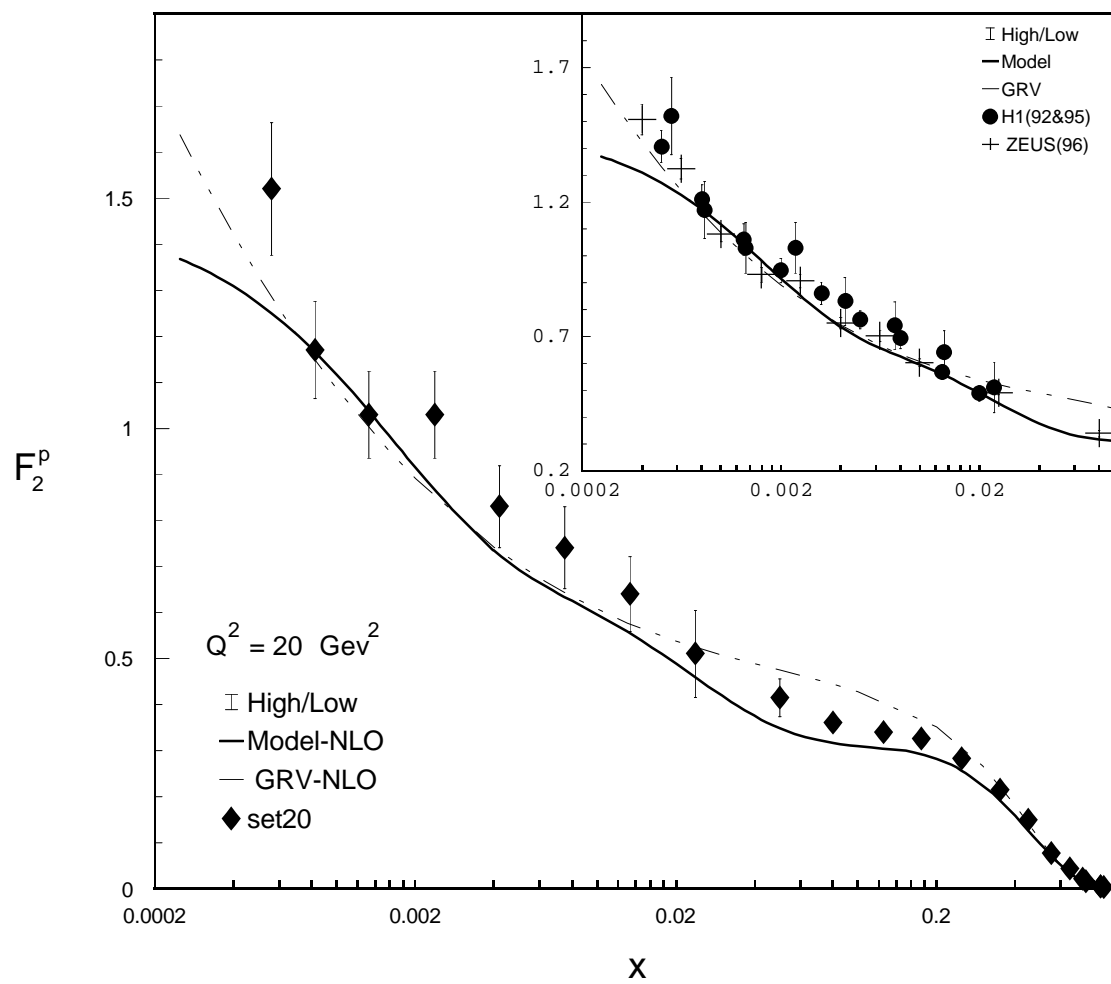


Fig. 6

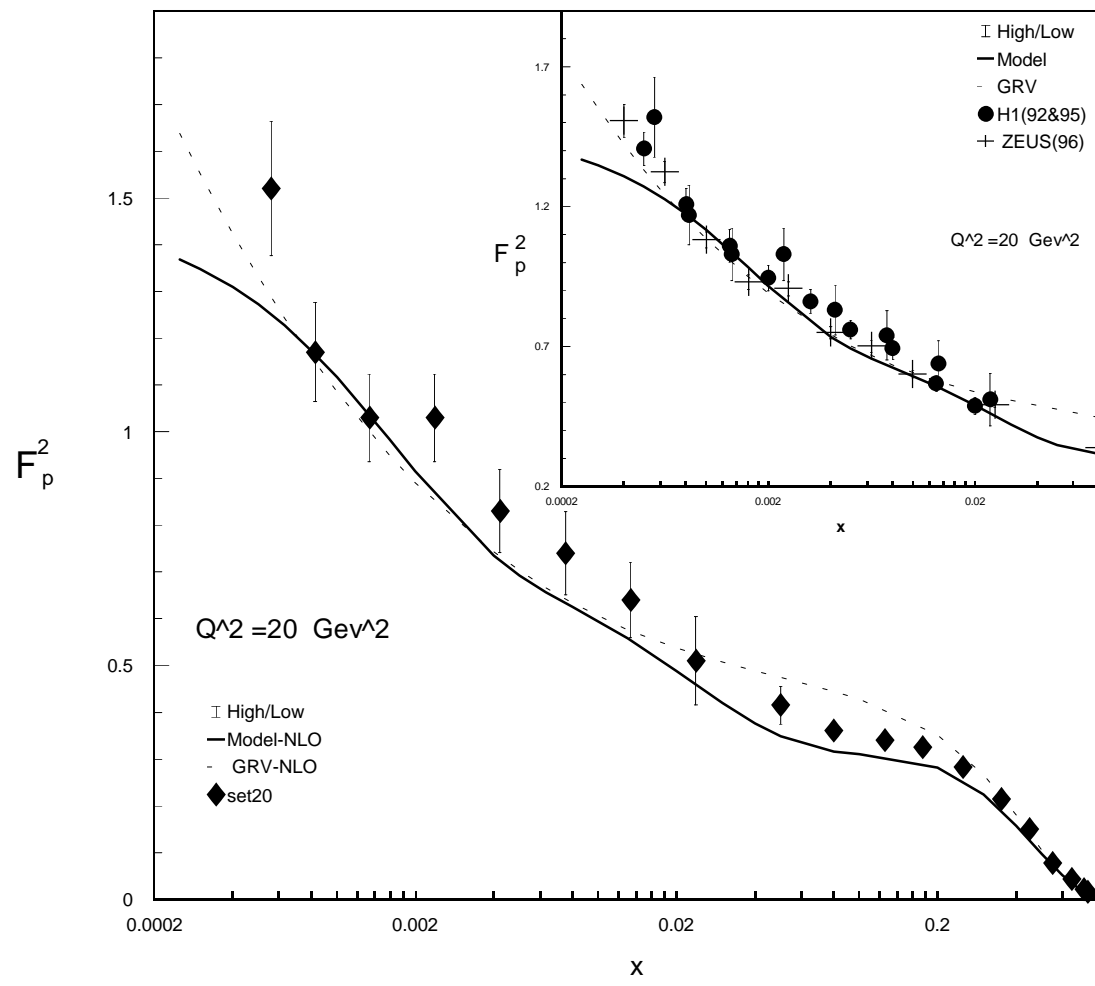


Fig.6

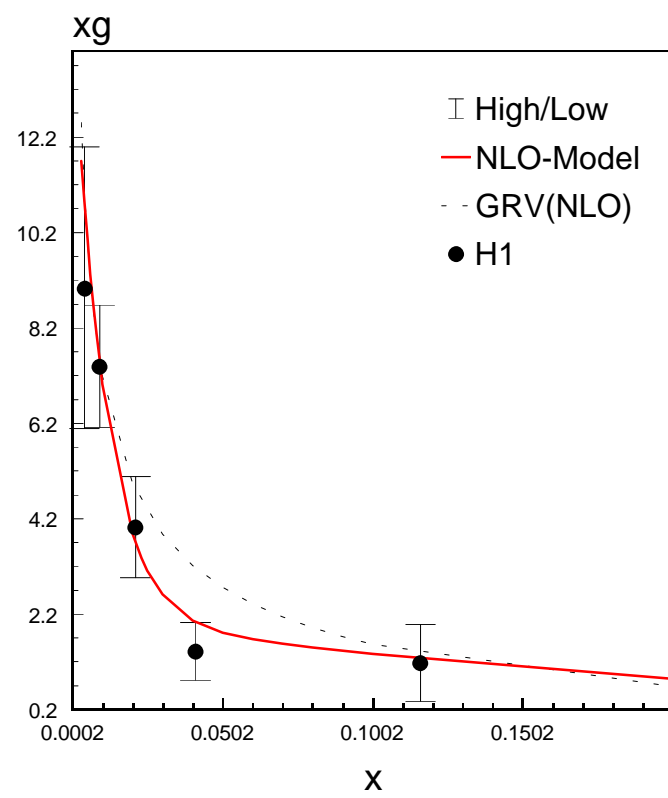


Fig.7

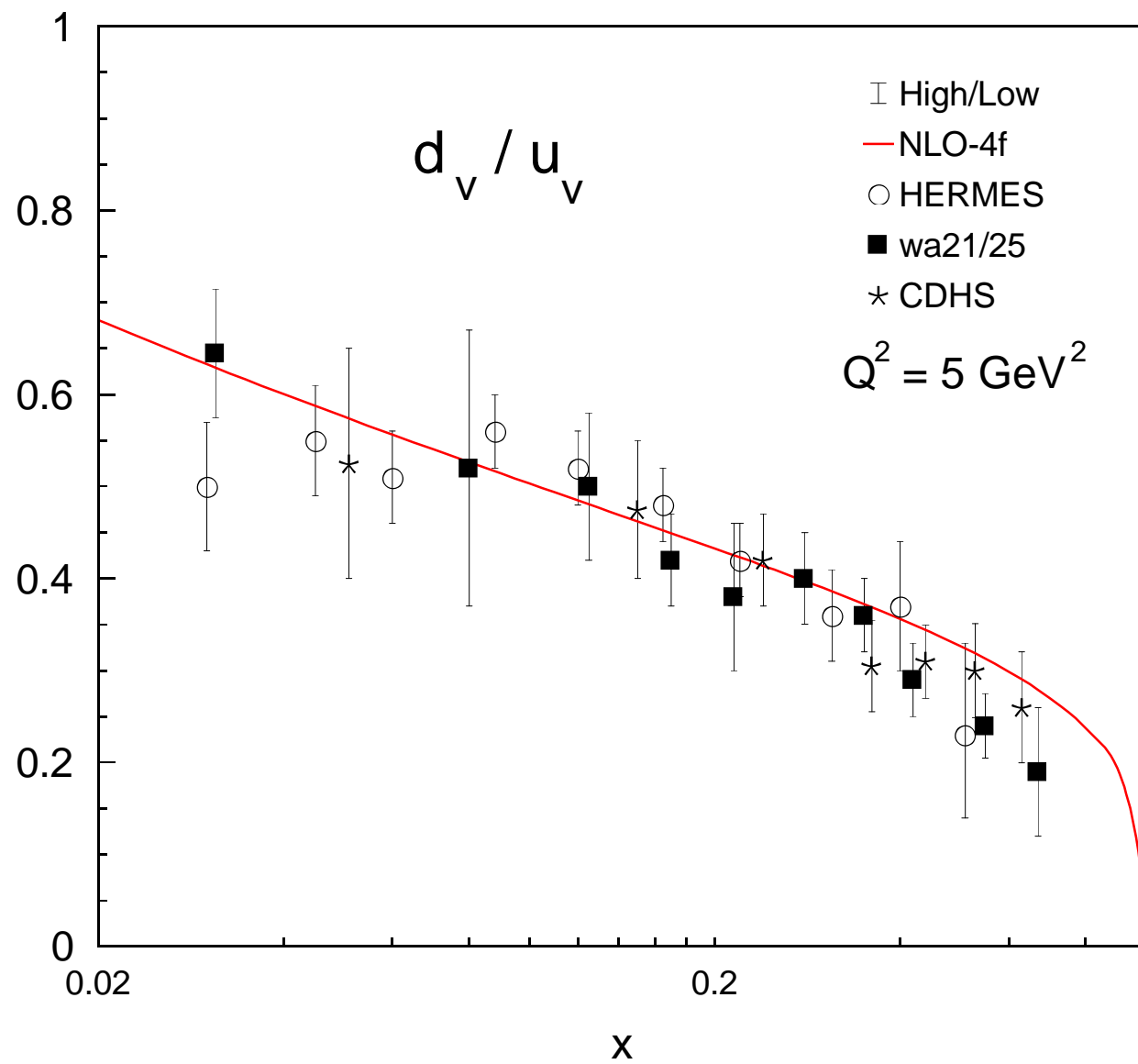


Fig. 8